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Eximia Journal
Vol. 6, 1-5, January, 2023
ISSN: 2784-0735
www.eximiajournal.com

Alternative proof of the Fermat's last Theorem

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Abstract. The Fermat's last theorem, has attracted the attention of mathematicians for more than three centuries. Many clever methods have been devised to attack the problem, and many beautiful theories have been created with the aim of proving the theorem. Yet, despite all the attempts, the question remains unanswered. The first successful proof was released in 1994 by Andrew Wiles and formally published in 1995 with over 100 pages. But I don't think it would not be the original un-written proof by Fermat, the proof should be much simple. Maybe people were sidetracked to other directions. Thing should go directly to the question without changing any of the conditions nor adding addition conditions. The paper introduces a re-defining the problem and based on the Fermat equation for n=2 as the foundation of the proof, follow by induction approach and finish the proof.

Keywords. vk function

A. Introduction

Fermat's Last Theorem states that no three positive integers x, y, and z satisfy the equation:

$$x^n + y^n = z^n$$

for any integer value of n greater than 2. The cases n = 1 and n = 2 have been known since antiquity to have infinitely many solutions.

The proposition was first stated as a theorem by Pierre de Fermat around 1637 in the margin of a copy of Arithmetica. Fermat added that he had a proof that was too large to fit in the margin. Although other statements claimed by Fermat without proof were subsequently proven by others and credited as theorems of Fermat, Fermat's Last Theorem resisted proof, leading to doubt that Fermat ever had a correct proof. Consequently, the proposition became known as a conjecture rather than a theorem. After 358 years of effort by mathematicians, the first successful proof was released in 1994 by Andrew Wiles and formally published in 1995.

B. Statement of problem

For z > y > x all are non-zero positive integers, there is no integer-triples (x, y, z) such that: $x^n + y^n = z^n$ for n>2.

As z > y> x all are non-zero positive integers, the equation can be rewritten as:

$\frac{x^n}{z^n} + \frac{y^n}{z^n} = 1$

Eximia Journal
Vol. 6, 1-5, January, 2023
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A function (hereinafter referred as VK function) is defined as:
$f(n) = \frac{x^n}{z^n} + \frac{y^n}{z^n}$
has no integer-triples solution(s) $\forall$ z > y> x > 0, where n > 2.

C. Observations

Now we explore the VK function by taking different values of n as follows:

When n = 0, f(n) = 2 $\neq$ 0, $\forall$ z > y> x

When n =1, $f(1) = \frac{x}{z} + \frac{y}{z}$, and there are infinite integer-triples that can satisfy the linear equation x + y = z or alternatively $(\frac{x}{z} + \frac{y}{z}) = 1$ in Diophantine equation form, for some z > y> x > 0
Example $\frac{2}{5} + \frac{3}{5} = 1$

When n = 2, $f(2) = \frac{x^2}{z^2} + \frac{y^2}{z^2}$ and there are infinite integer triples that were proven to satisfy the above equation, for example: $\frac{3^2}{5^2} + \frac{4^2}{5^2} = 1$ ….
When n = 3 or onwards, these are the cases that we want to prove that no integer triples is possible to satisfy the following equations:
$\frac{x^3+y^3}{z^3} = 1$
$\frac{x^4+y^4}{z^4} = 1$ …
$\frac{x^n+y^n}{z^n} = 1$….
$\frac{x^\infty+y^\infty}{z^\infty} = 1$
Obviously when n =∞, we have $\lim_{n\to\infty} \frac{x^n}{z^n} = 0$, and $\lim_{n\to\infty} \frac{y^n}{z^n} = 0$ $\forall$ z >x > y;
$f(n) = \frac{x^n}{z^n} + \frac{y^n}{z^n} = 0 \neq 1$ hence there is no integer-triple that can satisfy the equation $f(n) = \frac{x^n}{z^n} + \frac{y^n}{z^n} = 1$
Base on the results from previous scholars for n=2, we now start our proof below.

D. The Method

In part C above, it was showed that integer-triples (x, y, z) existed for n = 1 and n=2; implying that the necessary condition to meet is $f(n) = \frac{x^n}{z^n} + \frac{y^n}{z^n} = 1$
As $f(2) = \frac{x^2}{z^2} + \frac{y^2}{z^2}$ was proven to have integer-triple(s) with functional value 1,
If we can prove $f(3) = \frac{x^3}{z^3} + \frac{y^3}{z^3} \neq 0$ $\forall$ z >x > y; then we confirm f(3) has no integer-triple.
Therefore, we take the credit that f(2) = 1 as the base of the proof below:

As f(2) = 1 (i.e. Pythagorean triples), we have :
$f(3) - f(2) = (\frac{x^3}{z^3} + \frac{y^3}{z^3}) - (\frac{x^2}{z^2} + \frac{y^2}{z^2}) = (\frac{x^3}{z^3} - \frac{x^2}{z^2} + (\frac{y^3}{z^3} - \frac{y^2}{z^2})$
$= \frac{x^2}{z^2}(\frac{x}{z} - 1) + \frac{y^2}{z^2}(\frac{y}{z} - 1) < 0$ that implies f(3) < f(2),

Eximia Journal
Vol. 6, 1-5, January, 2023
ISSN: 2784-0735
www.eximiajournal.com

hence f(3) < 1, that no integer-triple(s) exists in f(3).

Similarly, we can repeat the proof of f(3) to f(4), f(5)….f(k)… but we are not prepared to do so one by one till infinite, instead if we can prove that the VK function is a decreasing function over n=2 to n =∞, then the whole proof of the Fermat's last Theorem is completed.

To prove that VK function is decreasing from n = 0 to n =∞,
We take f(n+1) – f(n) ∀ z > y> x

$$f(n+1) - f(n) = \left(\frac{x^{n+1}}{z^{n+1}} + \frac{y^{n+1}}{z^{n+1}}\right) - \left(\frac{x^n}{z^n} + \frac{y^n}{z^n}\right) = \left(\frac{xx^n}{zz^n} - \frac{x^n}{z^n}\right) + \left(\frac{yy^n}{zz^n} - \frac{x^n}{z^n}\right)$$

$$= \frac{x^n}{z^n}\left(\frac{x}{z} - 1\right) + \frac{x^n}{z^n}\left(\frac{y}{z} - 1\right)$$

as $\left(\frac{x}{z} - 1\right) < 0$ and; $\left(\frac{y}{z} - 1\right) < 0$, hence f(n+1) – f(n) < 0

As VK function is decreasing from n = 2 to n =∞, f(2) = 1
any subsequent f(n >2) must be less than 1. Hence as z > y> x, all are non-zero positive integers, the equation:
$\frac{x^n}{z^n} + \frac{y^n}{z^n} = 1$ has no solution(s), where n > 2.

E. Comments and summary

The first successful proof was released in 1994 by Andrew Wiles and formally published in 1995 with over 100 pages for which original un-written proof by Fermat was believed to have not more than a page. I refused to believe that Fermat never had a correct proof and "Fermat added that he had a proof that was too large to fit in the margin." it becomes my motivation to find out a much simple solution to this long outstanding problem.

N	f(n)	Example	Solution(s)	Functional form
0	$\frac{x^0}{z^0} + \frac{y^0}{z^0}$	$\frac{9^0}{111^0} + \frac{10^0}{111^0} = 2$	Trivial.	Constant.
1	$\frac{x}{z} + \frac{y}{z}$	$\frac{2}{5} + \frac{3}{5} = 1$	Infinite.	indeterminate equation x + y =z
2	$\frac{x^2}{z^2} + \frac{y^2}{z^2}$	$\frac{3^2}{5^2} + \frac{4^2}{5^2} = 1$	Infinite.	Pythagorean function $x^2 + y^2 = z^2$
3	$\frac{x^3}{z^3} + \frac{y^3}{z^3}$	$\frac{2^3}{59^3} + \frac{58^3}{59^3} < 1$	No solution.	$x^3 + y^3 = z^3$
..				
n	$\frac{x^n}{z^n} + \frac{y^n}{z^n}$	$\frac{1^n}{5^n} + \frac{3^n}{5^n} < 1$		$x^n + y^n = z^n$
…				
+∞	$\frac{x^\infty}{z^\infty} + \frac{y^\infty}{z^\infty}$	$\frac{1^\infty}{5^\infty} + \frac{3^\infty}{5^\infty} = 0$		Constant.

F. Findings and extension for further study

By means of the decreasing characteristic of the Fermat function: f(n+1) < f(n), the following findings are observed.
Findings:
1. For $f(1) = \frac{x}{z} + \frac{y}{z}$ = Sin α + Cos α = 1 which is an indeterminate equation. It represents

Eximia Journal
Vol. 6, 1-5, January, 2023
ISSN: 2784-0735
www.eximiajournal.com

different lines in a x-y plane by taking different z values. f(1) assumes universal solutions and can take any value, the value 1 is only one of the particular solutions.

2. For $f(2) = \frac{x^2}{z^2} + \frac{y^2}{z^2} = 1$ we observe that it represents an unit circle (in x-y plane) with $\mathrm{Sin}^2\alpha + \mathrm{Cos}^2\alpha = 1$, the solution (x, y) is on the circumference (z=1).

3. By means of the decreasing nature of the VK function, f(n+1) < f(n); it implies: $(\mathrm{Sin}^n\alpha + \mathrm{Cos}^n\alpha) < (\mathrm{Sin}^{n-1}\alpha + \mathrm{Cos}^{n-1}\alpha)$

Extensions:

1. For n ϵ Z, taking f(-1) which is an inverse of the function, we have

$f(-1) = \frac{x^{-1}}{z^{-1}} + \frac{y^{-1}}{z^{-1}} = \frac{z}{x} + \frac{z}{y}$ noting that $\frac{z}{x} > 1$ and $\frac{z}{y} > 1$ ∀ z > y> x,

it implies that $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$ has no positive integer triple ∀ z > y> x

There are some triples (x, y, z) = (4, -20, 5) or (4, 5, $\frac{20}{9}$) but none of these meet the condition z > y> x > 0

Similarly for $f(-2) = \frac{x^{-2}}{z^{-2}} + \frac{y^{-2}}{z^{-2}} = \frac{z^2}{x^2} + \frac{z^2}{y^2} > 1$;

it implies that $\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{z^2}$ has no positive integer triple ∀ z > y> x

2. For n ϵ Q > 0

By taking n = 1/2, we have

$f(\frac{1}{2}) = \sqrt{\frac{x}{z}} + \sqrt{\frac{y}{z}}$, infinite integer-triples existed at $f(\frac{1}{2})$, such as:

(4, 9, 25) ↔ ($\sqrt{\frac{4}{25}} + \sqrt{\frac{9}{25}} = 1$)

(25, 36, 121) ↔ ($\sqrt{\frac{25}{121}} + \sqrt{\frac{36}{121}} = 1$)

It is not difficult to get two quadric numbers (a, b) which sum is equal to another quadric number (c).

Weather the decreasing nature of the VK function holds between 0 and 1?

Checking is done below:

For $\frac{1}{2} < n < 1$, we have: $f(1) - f(\frac{1}{2})$

$= (\frac{x}{z} + \frac{y}{z}) - (\sqrt{\frac{x}{z}} + \sqrt{\frac{y}{z}}) = \sqrt{\frac{x}{z}}(\sqrt{\frac{x}{z}} - 1) + \sqrt{\frac{y}{z}}(\sqrt{\frac{y}{z}} - 1) < 0$

i.e. $f(\frac{1}{2}) > f(1)$ So the Fermat function is decreasing, for any value, say $f(\frac{4}{5})$ will have no integer triple.

For $0 < n < \frac{1}{2}$, we have $f(\frac{1}{2}) - f(0) = (\sqrt{\frac{x}{z}} + \sqrt{\frac{y}{z}}) - 2$ is negative, that is decreasing and it is also valid for $f(\frac{1}{5})$ that has no integer triple.

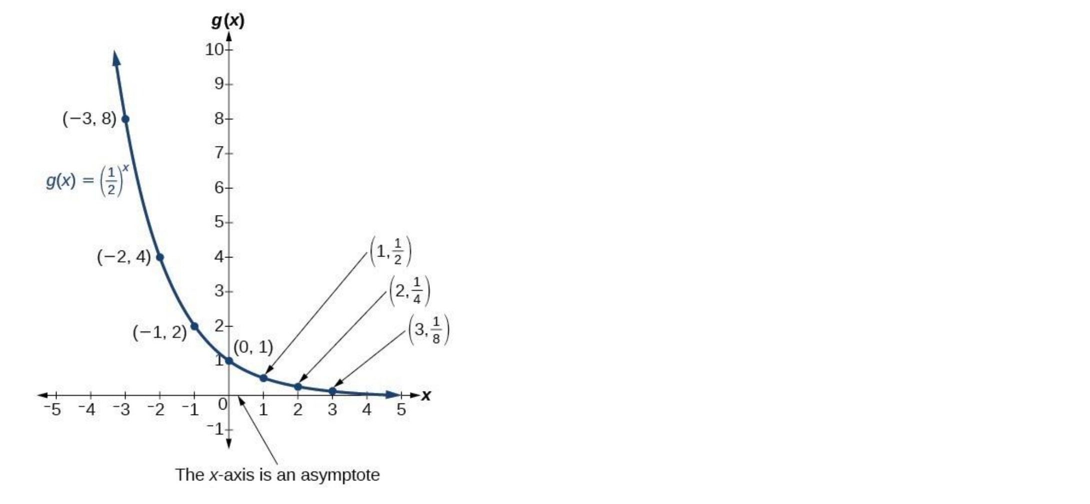
(Important note: the extension made is on discrete approach for value of n.)

G. Graphical representation

The VK function is the sum of 2 exponential functions of power n:

$f(n) = \frac{x^n}{z^n} + \frac{y^n}{z^n}$, such that ∀ z > y> x

Taking $g(\frac{1}{2})^n$ as example, the graph is presented below. Adding 2 exponential functions of power n is just shifting the graph upward which passed through (0, 2).



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