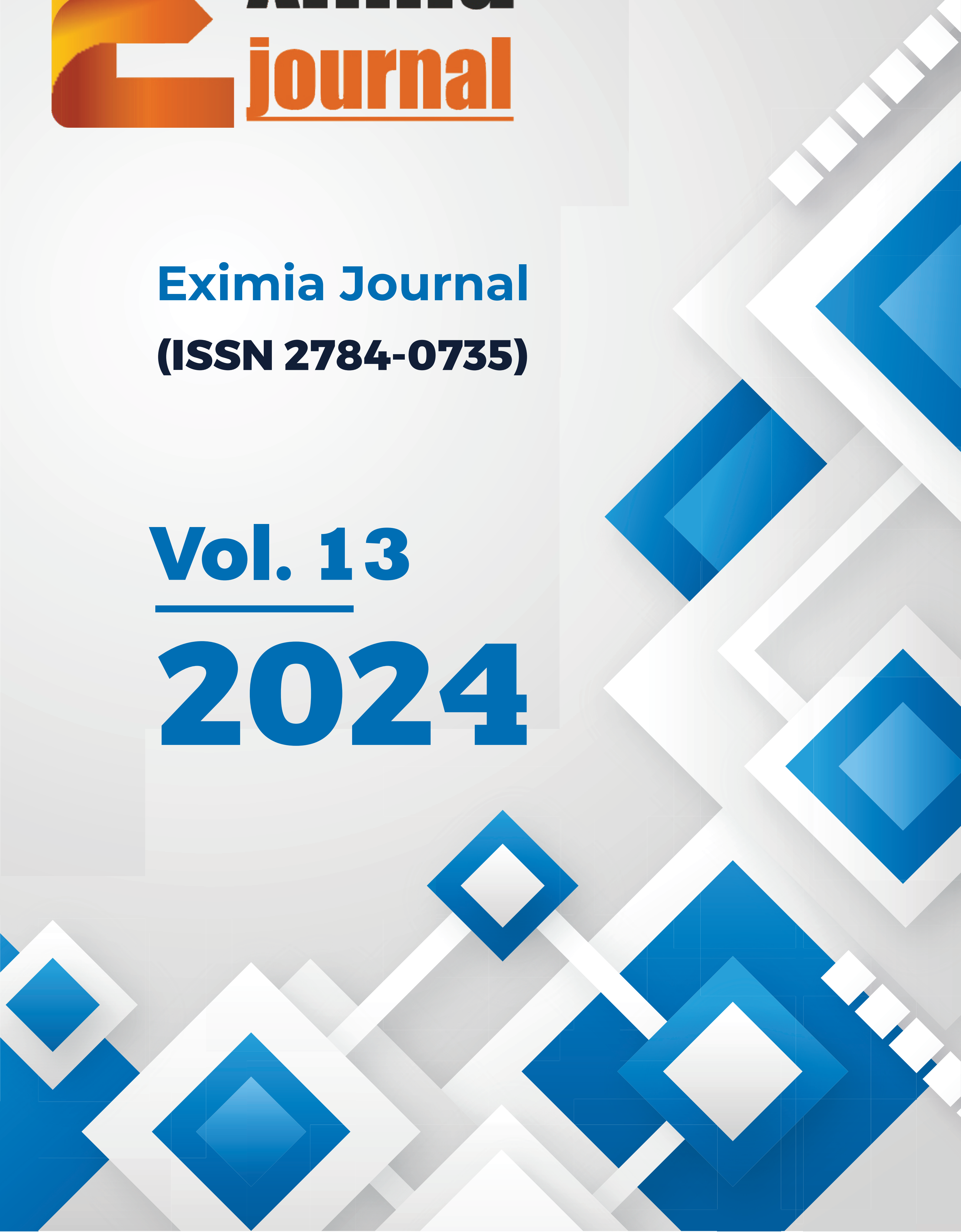
Eximia journal

Eximia Journal

(ISSN 2784-0735)

Vol. 13

2024



Eximia Journal
Vol. 13, 526-539, January, 2024
ISSN: 2784-0735
www.eximiajournal.com

Optimization of cut-off grades and depletion rates by means of dynamic programming

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Abstract. The determination of cut-off grades is critical for mine appraisal. Because of the time value of money, an optimum schedule of cut-off grades must be used instead of determination of a single cut-off grade for the life of a mine. In order to achieve the maximum net present value, a decreasing order of cut-off grades schedule must be used because of the opportunity cost. In the optimization of cut-off grades by dynamic programming, the depletion rates to be searched are actually limiting the search area. The optimum can be found from among the alternative depletion rates. Selling prices of mining products are volatile and future values cannot be known. This paper describes the use of dynamic programming in cut-off grades optimization and further extension of the method to price volatility in selling prices of mining products. The author introduces the general background of use of dynamic programming in cut-off grades optimization and the case in uncertain market conditions. Software developed in this subject handles to find the optimum sequence of cut-off grades and depletion rates for mineral deposits under market uncertainty. Dynamic programming method can be used as a means of cut-off grades optimization in this context very robustly.

Keywords. dynamic programming; cut-off grades; optimization; market uncertainty

1. INTRODUCTION

Optimization of cut-off grades is a fundamental issue for mineral deposits as it assigns the boundaries between ore and waste over time. The traditional approach to cut-off grades is to use the break-even grade, at which revenue equals cost. This approach completely ignores the time value of money and usually leads to a sub-optimal valuation of the mineral resource. The time value of money, together with the opportunity cost of taking low grades when higher grades are still available, ensures that there is an advantageous trade-off between mining at higher cut-off grades in the early years and lower cut-off grades in the later years of mine life. In order to achieve the maximum net present value, a decreasing order of cut-off grades schedule must be achieved because of the opportunity cost [1 – 5].

Mining ventures generally need long time for development works before starting production phase. Because of the long time span from the feasibility studies to the actual production, the costs and prices may change substantially that can affect the optimum cut-off grades policy.

Among the inputs for a cut-off grade optimization procedure, the selling price of the product of a mine is the most volatile. Any change in the selling price of a mining product in the global market makes every economic valuation obsolete. Because it is not possible to find the best possible revenue without finding the best cut-off grades scheme, uncertain selling prices (price fluctuations) of a mining product must be added to the algorithm for finding cut-off grades to be used. Meagher et al. used real options valuation for jointly handling geological and market uncertainties in open pit mine planning [6].

Dynamic programming is a multi-stage decision process that was developed by Bellman [7]. It has been applied to a number of mining problems including optimization of cut-off grades and depletion rates. Roman [8] introduced dynamic programming for determination of production rates. Dowd [2] extended it to optimize both production rates and cut-off grades.

Dynamic programming can be used for determining optimal sequences of cut-off grades in this context. In this method, every possible series of cut-off grade decisions are enumerated and an optimal solution is found by considering the effects of each decision separately.

Dowd [2] used dynamic programming technique to determine an optimal sequence of cut-off grades and production rates over the life of a mine. In this method, decision intervals must be set for cut-off grades and production rates and the full range of values is replaced by sets of discrete values defined by the intervals. For each depletion rate, a cut-off grade that gives maximum profit is calculated. The number of depletion rates restricts the number of possible solutions to be searched. The depletion rates are searched over the mine life by means of dynamic programming to find the maximum discounted profit, which gives a decreasing order of cut-off grades.

2. APPLICATION OF DYNAMIC PROGRAMMING TO CUT-OFF GRADES OPTIMIZATION

Roman [8] introduced dynamic programming to determine production rates, Dowd [2] extended it to optimize both production rates and cut-off grades and Cetin and Dowd [9] extended it to get optimum cut-off grades for multi-mineral deposits.

Multistage decision processes and the techniques of dynamic programming are the most conveniently introduced by discussing an example of a multistage decision process. The following is an example to show how dynamic programming can be applied to cut-off grades optimization problems.

Consider a small deposit that can be mined at two different cut-off grades. The maximum production rate of the mine is 500 tons a year. Assume that total amount of ore available is 4 000 tons if the lower cut-off grade is used. The deposit will be depleted in eight years since the maximum production rate of the mine is 500 tons a year. On the other hand, 2 000 tons of ore is available at the higher cut-off grade and the deposit would be depleted in four years at this cut-off grade. Suppose that 1 ton of ore at the lower grade can be sold for £10 and 1 ton of ore at the higher grade can be sold for £16. The discount rate is 10%.

The problem here is to find the optimal cut-off grade policy that gives the maximum net present value at the end of depletion of the deposit. The possible decisions are illustrated in Figure 1. Since the problem is a very small scale and simplified one, it is possible to solve it graphically step by step. It should be understood that this is a very elementary example intended only to demonstrate the basic concepts of dynamic programming.

This is an example of discrete deterministic stationary discounted dynamic programming. Because of the nature of the problem, there is a starting point but there are many end points. In other words, there is a group of possible final points on the upper horizon of the figure, which represent depletion of the deposit.

In the first year, either the deposit will be exploited by applying a lower cut-off grade, indicated by the point A, or the higher cut-off grade leading to the state indicated by the point B. If the lower cut-off grade is applied, one-eighth of the deposit would be mined and the discounted value of the first year's profit would be £4545. However, if the higher cut-off grade is applied, one-fourth of the deposit would be mined and the discounted value of the first year's profit would be £7273. As a result, we can write the value £4545 next to point A and the value £7273 next to point B, each representing the value of the routes to these points.

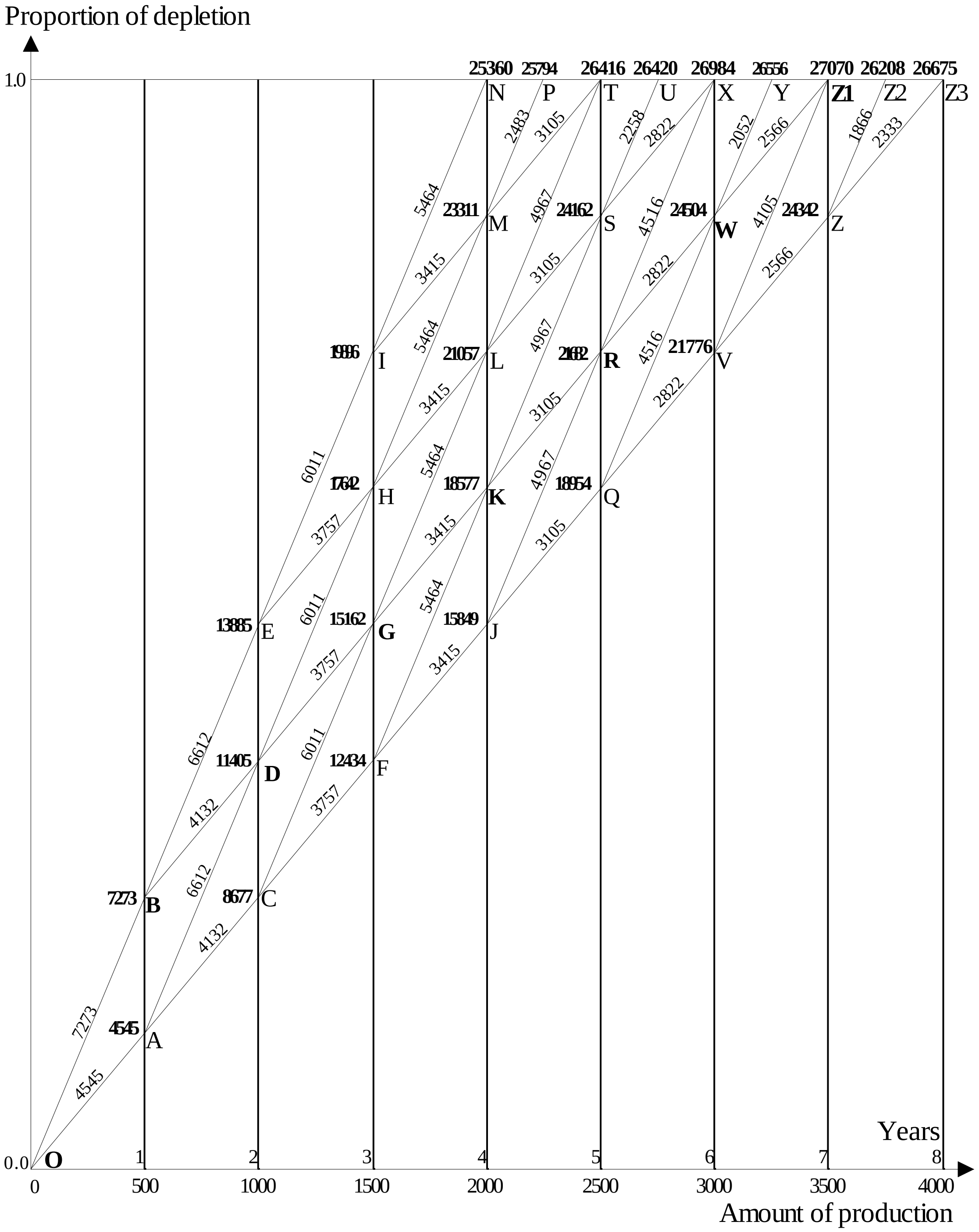

Fig. 1. Decision graph for a cut-off grades problem solved by dynamic programming

In the next stage of the computation, the states of the system are indicated by the points C, D and E. In order to find the return values of these points, one must look for the routes from the point O to C, O to D and O to E. But as a consequence of the principle of optimality, it is only necessary to consider the routes from the points A and B, since they have already been assigned optimal values. For the point C, there is only one route and it is OAC. As a result, one can write £8677 as the value of arriving at point C in an optimal manner; this value is obtained by adding the values of point A to the value of the decision AC. For point E, there is also only one route and it is OBE. Therefore, one can write £13885 as the optimum value of the second year's profit for that point as the sum of the values of state B and the decision BE. However, for D, there are two routes to be considered: OAD and OBD. The value of the route OAD is £11157 and the value of the route OBD is £11405. Route OBD has the higher value and thus the optimum value of the second year's profit for state D is £11405. For the third year, the optimal routes from the starting point O to the points F, G, H and I are searched to find the optimal values of profits of these states.

When doing this, it is sufficient to only take into account the values of the points C, D and E, in the previous year and the routes starting from these points.

This process is continued until the upper depletion horizon is reached. At the end of the process, the highest value among the return values of the points on the upper horizon of the figure gives the maximum profit. That point is Z1. Consequently, the route that leads to the maximum value is the optimal route. The optimal route could be found by tracing back easily. The optimal route is OBDGKRWZ1, which means mining at the higher cut-off grade during the first year and thereafter mining at the lower cut-off grade. The maximum net present value of this policy is £27070.

An expression for this principle of optimization can be formulated as follows:

Let

$$R(x_1, y_1; x_2, y_2; \dots\dots\dots; x_n, y_n) = g_1(x_1, y_1) + g_2(x_2, y_2) + \dots\dots\dots\dots + g_n(x_n, y_n) \qquad (1)$$

where;

R: total return,
n: period,
x_n: reserve amount in period n,
y_n: grade percentage in period n,
g_n: return in period n.

The objective is to maximise the return R subject to the constraints

$x_i \geq 0$ and

$$\sum_{i=1}^{n} xi = x$$

where;

x: total ore reserve.

a. The basic functional equation for this maximisation is

$$f_n(x) = \max_{x_n, y_n}[g_n(x_n, y_n) + f_{n-1}(x - x_n)] \qquad (2)$$

where;

f_n: the return for n periods.

3. UNCERTAIN MARKET CONDITIONS

Among the inputs for a cut-off grade optimization procedure, the selling price of the product of a mine is the most volatile. Any change in the selling price of a mining product in the global market make every economic valuations obsolete. One must revaluate the plans according to the current market conditions. Basically, nobody knows how the prices will change in the future, but can estimate. All the sensitivity and risk analysis are done in order to inform the entrepreneurs for estimated risks. However, although risk factor gives an idea to the entrepreneurs about the risks of a mining establishment, it does not affect the optimality calculations. But the optima has to be rearranged after market price changes.

Although sensitivity and risk analysis are good tools for decision making in entrepreneurship, they do not give any clue for the optimum cut-off grades policy for stochastically estimated volatile market conditions. In order to make a better mining plan, one must solve the problem of annual dynamic cut-off grades optimally, and that necessitates taking the volatile market conditions into consideration. This is not related to decision making about accepting or rejecting a project. But just to find the best cut-off grades as accurately as possible. Therefore, it is accepted in this work that uncertain selling prices of a mining product must be added to the algorithm for finding cut-off grades to be used. Because, without finding the best cut-off grades scheme it is not possible to find the most revenue. And using today's selling price of a commodity, instead of estimated future values of that, may sacrifice to reach the best net present value.

In order to forecast the future one must apply the past. Past market prices give identification for the future price estimations. Estimations can be deterministic or stochastic by nature. More weight must be given to the recent years than to the previous years.

4. OPTIMIZATION OF CUT-OFF GRADES BY MEANS OF DYNAMIC PROGRAMMING UNDER MARKET UNCERTAINTY

In the optimization of cut-off grades by dynamic programming, the depletion rates to be searched are actually limiting the search area. In the determination of cut-off grades and depletion rates by dynamic programming, the rates of depletion are defined by the tonnage interval on which a decision to mine or not can be made. The optimum for each year can be found from among the alternative depletion rates. Basic equations for this principle of optimization can be found in equations 1 and 2.

The type of dynamic programming technique used in this work is discrete, deterministic and stationary discounted.

The following expressions specify the numbers of calculations involved in the programs developed in this work.

The number of different depletion rates is:

$$d = \frac{M}{I} + 1 \qquad (3)$$

where;

d: number of possible depletion rates,
M: maximum depletion rate,
I: tonnage interval.

The number of different depletion rates above the lowest depletion rate is:

$$D = d - \frac{H}{I} \qquad (4)$$

where;

D: number of different depletion rates above the lowest depletion rate,
H: maximum production rate (or minimum depletion rate).

The number of stages of the system is:

$$n = \frac{T}{H} + 2 \qquad (5)$$

where;

n: number of stages,
T: total reserve in the system.

If the stages are 1,2,3,…..,n-2,n-1,n, the number of states for each stage is:

$$s_n = n * (D - 1) + 1 \qquad (6)$$

where;

s_n: number of states for each stage

The total number of calculations in searching for all possible combinations is:

$$\textit{total}\ \text{number of calculations} = \sum_{m=1}^{n} D * s_m \qquad (7)$$

Selling price is added to the algorithm stochastically. Exponential increments are used in order to add selling prices to the algorithm. Each increment has equal weight in the algorithm. Later, Monte Carlo Simulation takes place to choose the estimated value. The total number of incremental years to be added to the algorithm can be formulated as follows:

$$Y = \sum_{k=0}^{n} 2^k \qquad (8)$$

where;

Y : number of past years to be included.
n : number of increments.
2^0 : price of this year.

One safeguard to be taken in adding selling prices to the process is that, fixed (real) prices are used instead of current (nominal) ones. The effect of the inflation of the money used must be eliminated explicitly.

At last, the average value of simulated selling prices are added to the algorithm. After the inclusion of stochastic prices to the algoritm, Lane [1]'s equation for cut-off grades becomes:

$$v = (\mu_\delta - k) * x * y * a - x * h - m - (f + F) * t \qquad (9)$$

where;

v: net present value,
μ_δ: mean of stochastic selling prices per unit of marketed product,
p: price per unit of marketed product,
k: marketing variable cost,
x: ore/material ratio,
y: yield from treatment (recovery),
a: average grade,
h: mineral processing variable cost,
m: mining variable cost,
f: fixed cost,
F: opportunity cost,
t: time per unit of resource.

The following expressions are derived for cut-off grade calculations:

Assume that the grade-tonnage distribution of a mineral deposit consists of W grade cells for mineral 1. Hence, there would be $W + 1$ grade limits. The representation of the corresponding grade for the different cells would be:
$[g_1(1), g_1(2)], [g_1(2), g_1(3)], \ldots, [g_1(W - 1), g_1(W)], [g_1(W), g_1(W + 1)]$
If the lower grade limit g(w) for a given cell $[g(w), g(w + 1)]$ is the cut-off grade representing interval p, the amount of the material above the cut-off grade, the amount of the material below the cut-off grade, the average grade above the cut-off grade can be found by using the following equations:

$$T_{ore}(p) = \sum_{w=1}^{W} T_p \qquad (10)$$

$$\text{where} \quad T_p = \begin{cases} T_{(w)} & \text{if } W \geq w \geq p \text{ for } \forall w \in [1, W] \\ 0 & \text{otherwise} \end{cases}$$

where;
$T_{ore}(p)$: amount of material above the cut-off grade for the p^{th} grade interval,
$T_{(w)}$: amount of material for the given grade limits,
p: grade interval,

$$T_{waste}(p) = \sum_{w=1}^{W} T_p \qquad (11)$$

$$\text{where} \quad T_p = \begin{cases} T_{(w)} & \text{if } 0 \leq w \leq p \text{ for } \forall w \in [1, W] \\ 0 & \text{otherwise} \end{cases}$$

where;
$T_{waste}(p)$: amount of material below the cut-off grade for the p^{th} grade interval.

$$g_{avg}(p) = \frac{\sum_{w=1}^{W} T_p \times \left(\frac{g_w + g_{w+1}}{2} \right)}{\sum_{w=1}^{W} T_p} \qquad (12)$$

where

$$\begin{bmatrix} T_p \\ g_w \\ g_{w+1} \end{bmatrix} = \begin{cases} \begin{bmatrix} T_{(w)} \\ g_1(w) \\ g_1(w+1) \end{bmatrix} & \text{if } W \geq w \geq p \text{ for } \forall w \in [1, W] \\ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} & \text{otherwise} \end{cases}$$

where;
$g_{avg}(p)$: average grade above the cut-off grade for the p^{th} grade interval,
$g(w)$: lower grade limit of a given cell,
$g(w + 1)$: upper grade limit of a given cell.

The ore/material ratio can be found by using the Equation 4.

$$x(p)=\frac{T_{ore}(p)}{T_{ore}(p)+T_{waste}(p)} \qquad (13)$$

where;

$x(p)$: ore/material ratio for the p^{th} grade interval.

5. CASE STUDY

A case study has been included here to illustrate the application of the software for determining optimum cut-off grades for uncertain market conditions. The program is written in C++ code.
The case study is a copper deposit. The grade tonnage distribution for the deposit is shown in Figure 2 and the technical and economic data are given in Table 1. The computation results showing the complete cut-off grades policy are given in Table 2.
466 different cut-off grades and 61 different depletion rates for each mineral were searched for the optimum. The selling prices of previous years are real prices rather than nominal.
The mining operation terminates in 15.35 years. Total discounted profit is $3 716 990 000 while the average value of stochastic selling prices is $6 960. The program terminates in 1 CPU second.
For the sake of comparison, the same data are applied to the program written for cut-off grades optimization by means of dynamic programming without market uncertainty [10]. Current selling price of copper ($8 400) is used and the total discounted profit is $4 948 630 000. The program terminates in 1 CPU second.
The optimum cut-off grades change and the depletion rates decrease throughout the life of the mine for both of the methods. Discounted profits are very different because of different selling prices.
In the first case that the results are scheduled in the Table 2, the average of stochastically searched selling prices by means of Monte Carlo Simulation technique is used. The program written necessitates the simulation runs 1000 times and the mean value is considered as the average value of the stochastic selling prices.
In the second case that the results are scheduled in the Table 3, the selling price of the current year, that is, the first selling price ($8 400) in the Table 1 is used as is used in traditional approaches.

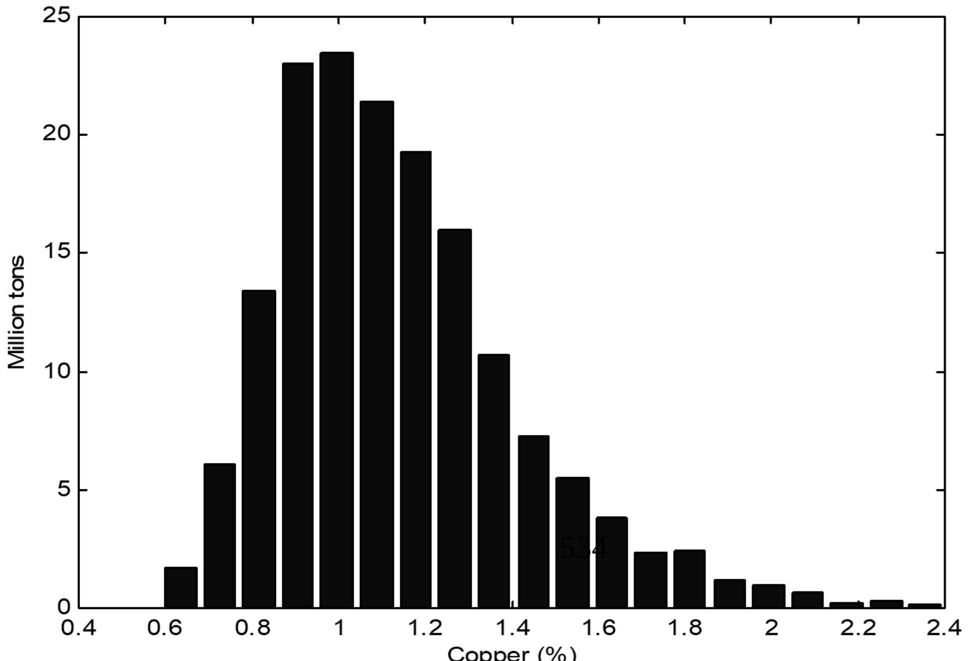

Table 1. Technical and economic data for the case study.

Description	Value
Lower limit of cut-off grades (%)	0.635
Upper limit of cut-off grades (%)	1.1
Interval between cut-off grade decisions (%)	0.001
Mining Capacity (tons per year)	13 000 000
Mineral Processing Capacity (tons per year)	10 000 000
Tonnage interval between decisions to mine or not (tons per year)	50 000
Marketing and/or refining capacity (tons per year)	130 000
Current selling price for copper (dollars per ton)	8400
Selling price for copper for the last year (dollars per ounce)	8375
Selling price for copper previous year (dollars per ounce)	9551
Selling price for copper previous year (dollars per ounce)	7772
Selling price for copper previous year (dollars per ounce)	6077
Selling price for copper previous year (dollars per ounce)	6075
Selling price for copper previous year (dollars per ounce)	6834
Selling price for copper previous year (dollars per ounce)	5660
Selling price for copper previous year (dollars per ounce)	4638
Selling price for copper previous year (dollars per ounce)	6454
Selling price for copper previous year (dollars per ounce)	7348
Selling price for copper previous year (dollars per ounce)	8224
Selling price for copper previous year (dollars per ounce)	7964
Selling price for copper previous year (dollars per ounce)	9948
Selling price for copper previous year (dollars per ounce)	7707
Selling price for copper previous year (dollars per ounce)	3418
Selling price for copper previous year (dollars per ounce)	7580
Selling price for copper previous year (dollars per ounce)	7854
Selling price for copper previous year (dollars per ounce)	5530
Selling price for copper previous year (dollars per ounce)	3939
Selling price for copper previous year (dollars per ounce)	2836
Selling price for copper previous year (dollars per ounce)	2097
Selling price for copper previous year (dollars per ounce)	1970
Selling price for copper previous year (dollars per ounce)	2549
Selling price for copper previous year (dollars per ounce)	2510
Selling price for copper previous year (dollars per ounce)	2145
Selling price for copper previous year (dollars per ounce)	2602
Selling price for copper previous year (dollars per ounce)	3420
Selling price for copper previous year (dollars per ounce)	4538
Selling price for copper previous year (dollars per ounce)	4766
Selling price for copper previous year (dollars per ounce)	2826
Variable mining cost of material mined (dollars per ton)	2.4
Variable concentration cost of material processed (dollars per ton)	9.6
Variable marketing and/or refining cost (dollars per ton)	1500
Fixed costs (dollars per year)	2 500 000
Recovery rate for copper (%)	92
Discount rate (%)	10

Table 2. Output file for the case study, with market uncertainty. Selling price is $6 960. Total discounted profit is $3 716 990 000.

Years	Cut-off grades (%)	Profit ($m)	Discounted profit ($m)	Depletion rate (Mt)	Production rate (Mt)	Marketing rate (kt)
1	0.932	503.90	458.09	12.10	10.00	115.64
2	0.904	496.86	410.62	11.50	10.00	114.09
3	0.857	489.61	367.85	10.95	10.00	112.52
4	0.848	488.14	333.41	10.85	10.00	112.21
5	0.803	481.86	299.20	10.45	10.00	110.89
6	0.781	480.10	271.00	10.35	10.00	110.52
7	0.747	477.27	244.92	10.20	10.00	109.93
8	0.720	475.27	221.71	10.10	10.00	109.52
9	0.635	472.96	200.58	10.00	10.00	109.06
10	0.635	472.96	182.35	10.00	10.00	109.06
11	0.635	472.96	165.77	10.00	10.00	109.06
12	0.635	472.96	150.70	10.00	10.00	109.06
13	0.635	472.96	137.00	10.00	10.00	109.06
14	0.635	472.96	124.54	10.00	10.00	109.06
15	0.635	472.96	113.22	10.00	10.00	109.06
16	0.635	165.54	38.33	3.50	3.50	38.17

The aim of including the results of both methods here is not to show that the method that include uncertain market conditions gives higher discounted profit. Because the selling price is lower ($6 960) in the first case, the discounted profit has to be lower. The aim is to show that the optimum cut-off grades schedule changes when the selling price changes and the cut-off grades sequence in the Table 2 must be used in order to achieve the maximum discounted profit.

Table 3. Output file for the case study, without market uncertainty. Selling price is $8 400. Total discounted profit is $4 948 630 000.

Years	Cut-off grade (%)	Profit ($m)	Discounted profit ($m)	Depletion rate (Mt)	Production rate (Mt)	Marketing rate (kt)
1	0.921	666.66	606.05	11.85	10.00	115.01
2	0.904	661.13	546.39	11.50	10.00	114.09
3	0.857	651.63	489.58	10.95	10.00	112.52
4	0.848	649.71	443.76	10.85	10.00	112.21
5	0.803	641.52	398.34	10.45	10.00	110.89
6	0.781	639.23	360.83	10.35	10.00	110.52
7	0.720	632.96	324.81	10.10	10.00	109.52
8	0.678	631.53	294.61	10.05	10.00	109.30
9	0.635	629.99	267.18	10.00	10.00	109.06
10	0.635	629.99	242.89	10.00	10.00	109.06
11	0.635	629.99	220.81	10.00	10.00	109.06
12	0.635	629.99	200.73	10.00	10.00	109.06
13	0.635	629.99	182.48	10.00	10.00	109.06

14	0.635	629.99	165.89	10.00	10.00	109.06
15	0.635	629.99	150.81	10.00	10.00	109.06
16	0.635	245.69	56.67	3.90	3.90	42.53

For the sake of comparison, the cut-off grades and depletion rates schedules of both cases (current and estimated selling prices) are shown in a single table and the cutoff grades schedule of them are shown in a figure (Table 4 and Fig. 3). In the case of lower selling price, cut-off grades remain higher that can be traced in Fig. 3. In the case of higher selling price, cut-off grades are lower.

Table 4. Cut-off grades and depletion rates schedule for different selling prices

	Selling price ($8 400)		Selling price ($6 960)	
Years	Cut-off grade (%)	Depletion rate (Mt)	Cut-off grades (%)	Depletion rate (Mt)
1	0.921	11.85	0.932	12.10
2	0.904	11.50	0.904	11.50
3	0.857	10.95	0.879	10.95
4	0.848	10.85	0.857	10.85
5	0.803	10.45	0.814	10.45
6	0.781	10.35	0.792	10.35
7	0.720	10.10	0.747	10.20
8	0.678	10.05	0.720	10.10
9	0.635	10.00	0.678	10.00
10	0.635	10.00	0.635	10.00
11	0.635	10.00	0.635	10.00
12	0.635	10.00	0.635	10.00
13	0.635	10.00	0.635	10.00
14	0.635	10.00	0.635	10.00
15	0.635	10.00	0.635	10.00
16	0.635	3.90	0.635	3.50

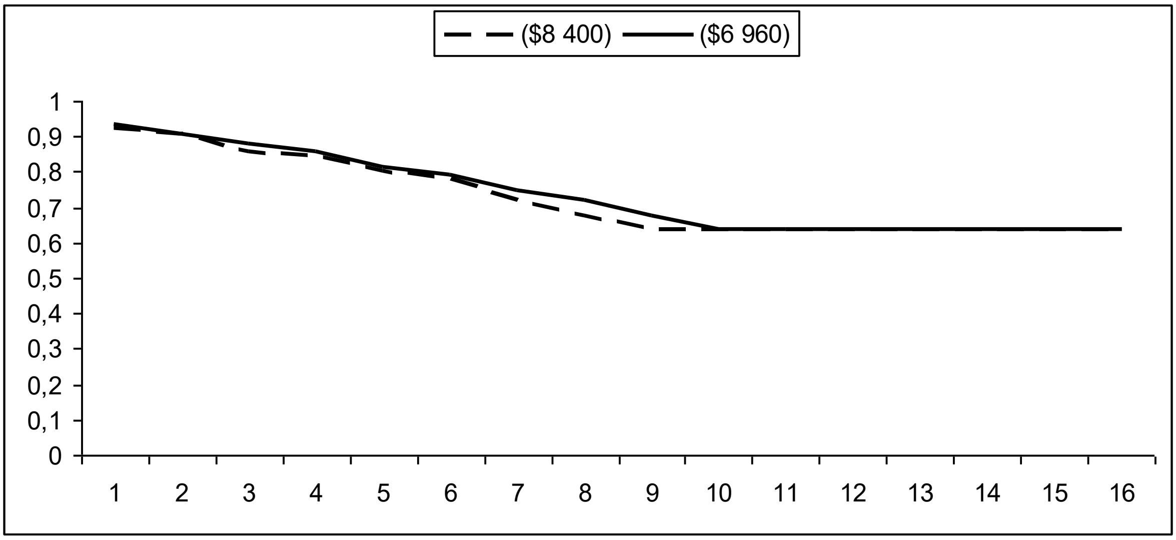

Fig. 3. Cut-off grades for different selling prices

6. CONCLUSIONS

The objective of the work described in this paper is to find a proper way of determining optimal production sequences of cut-off.

In this study, variable mining costs are applied to a depletion rate, which means that all the material required to achieve a specific tonnage and grade is effectively excavated regardless of whether it is processed or treated as waste material.

The computer program is based on dynamic programming method. The work described here extends the use of dynamic programming for cut-off grades optimization to that include market price uncertainty. Dynamic programming is a very robust search engine and it is very useful in cut-off grades and depletion rates optimization. After deciding different depletion rates to be searched for the optimum, it is very trivial for the program to find the global optimum in a few seconds.

Since market prices are uncertain, they must be estimated. Without doing that and using current selling prices of mining products leads unfruitful evaluations of mineral deposits. Therefore, the selling prices ought to be estimated for future before the mine appraisal takes place.

After the work done for this subject, it is very clear that it is not possible to find optimal cut-off grades without using estimated selling prices.

This research did not receive any specific grant from funding agencies in the public, commercial, ot not-for-profit sectors.

7. REFERENCES

[1] Lane KF. Choosing the Optimum Cutoff Grade, Colorado School of Mines Quarterly, 59, 811-829; 1964.

[2] Dowd PA. Application of dynamic and stochastic programming to optimise cut-off grades and production rates, Trans. Instn. Min. Metall., (Section A: Mining Industry), 81, 160-179; 1976.

[3] Cetin E, Dowd PA. Multiple cut-off grade optimization by genetic algorithms and comparison with grid search method and dynamic programming. Journal of the Southern African Institute of Mining and Metallurgy, 116(7), 681-688; 2016.

[4] Cetin E. Assessment of costs in cut-off grades optimization by using grid search method. Gospodarka Surowcami Mineralnymi, 34; 2018.

[5] Cetin E, Dalgic A. The optimization of cut-off grades by means of memetic algorithms. Gospodarka Surowcami Mineralnymi-Mineral Resources Management, 107-122; 2022.

[6] Meagher C, Abdel Sabour SA and Dimitrakopoulos R. Pushback design of open pit mines under geological and market uncertainties. Proceedings of the International Symposium on Orebody Modeling and Strategic Mine Planning: Old and New Dimensions in Changing World. Vol. 17, pp. 297-304); 2009.

[7] Bellman RE. Dynamic Programming, Princeton University Press; 1957.

[8] Roman RJ. The Use of Dynamic Programming for Determining Mine-mill Production Schedules, Proceedings of 10th Symposium on Applications of Computer and Operations Research in the Mineral Industry, Johannesburg, pp 165-170; 1973.

[9] Cetin E, Dowd PA. Multi Mineral Cut-off Grade Optimisation By Means of Dynamic Programming, Sustainable Production and Consumption of Mineral Resources, Krakow, Poland; 2011.

[10] Cetin E. Optimisation of Cut-off Grades for Multi-mineral Deposits, PhD Thesis, The University of Leeds Department of Mining and Mineral Engineering; 2001.