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# The Prediction of Rank Reversal in TOPSIS

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**Abstract**. The rank reversal (RR) problem occurs in that rank the all-around performance (benefit or cost) of some alternatives represented with respect to rivaling criteria. The RR in the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) occurs less than other multi-criteria decision-making (MCDM) methods. We analyze the RR of simplified TOPSIS by linear normalization method of mathematic expressions. Many past papers had researched the RR, and some studies used a virtual non-dominated alternative to being ranked first to avoid RR in the original set of alternatives. However, these given and fictious alternatives are less compelling in real world objects. In addition, some authors tried to resolve RR in specific situation of TOPSIS. Consequently, we propose a manner to analyze how the mathematic expressions of TOPSIS affects the rankings, thus appraising, predicating and avoiding the RR. In order to make clear the effect of the change on the performance in any criterion, we use a simplified TOPSIS under linear normalization in decision matrix. The variation of any criterion's component factor will be discussed and analyzed in the specified scenarios. Some examples are supplied to interpreted the mentioned effect. This research may could be the concept, recommendation and assistance for preventing and predicting the RR in TOPSIS.

**Keywords**. Rank Reversal, TOPSIS, MCDM, Non-dominated alternative

## 1. Introduction

In the past several decades, the multiple-criteria (attribute) decision-making (MCDM/MADM) are very popular in social science and natural science fields. The Technique for Order Preference by Similarity to the Ideal Solution (TOPSIS) is one of many methods in MCDMs. It is useful for solving and dealing with MCDM topics under discussion which is first proposed by Hwang and Yoon in 1981. Many other approaches are combined with TOPSIS to analyze many questions in practical issues. These methods include Analytic Hierarchy Process (AHP), fuzzy AHP, Data Envelopment Analysis (DEA), Analytic Network Process (ANP), etc. In 1983, Belton and Gear first identified and discussed the rank reversal (RR) problem in AHP, and Salo and Hämäläinen this problem to the relative measurement mode of AHP in 1997.

The RR means a inconsistent rank ordering of evaluation alternatives when the available alternatives changes (increase / decrease) or some criteria change in MCDM. Besides TOPSIS and AHP, many other MCDM methods also have the problem of RR, such as VIseKriterijumska Optimizacija IKOmpromisno Resenje (VIKOR), Evaluation based on

Distance from Average Solution (EDAS), ELimiation EtChoice Translating Reality (ELECTRE), Multiplicative Exponential Weighting (MEW), and Preference Ranking Organization Method for Enrichment of evaluations (PROMETHEE). Ren et al. (2007) proposed a novel M-TOPSIS method to avoid the RR, but it is lack of verification and validation on theorem. Wang and Luo (2009) explained the RR of TOPSIS by the counter examples, but there is no solution to settle or reduce the RR problem. García-Cascales and Lamata (2012) suggested two manners to reduce the RR of the TOPSIS method. (i) Establishing a new standard of normalization, but it is no enough to improve the RR problem. (ii) Both the positive ideal solution (PIS) and the negative ideal solution (NIS) are replaced by a absolute mode, but this manner is invalid on some cases. For example, it is invalid when the non-dominated cost criterion is replaced. Aires and Ferreira (2019) proposed a new approach, R-TOPSIS method, to avoid RR cases in the TOPSIS method. They corroborated the usefulness by statistical perspective, but there is lack of exposition and evidence in the proposed method. In additional to mentioned methods, some scholars proposed some coordinated MCDM methods to avoid RR, but they are weakened focus on the innate character analysis of RR. It is better way to analyze and explain the RR problem from a general perspective, and conclude the threshold of RR by some mathematical formulas.

De Farias Aires and Ferreira (2019) organized a literature review on RR and MCDM methodologies, including the TOPSIS. Five types of RR are: (i) That an irrelevant alternative is increased or reduced may cause rank ordering of alternatives inconsistent with the original rank. (ii) That a dominated alternative is replaced by the inferior alternative may cause the original best alternative changes. (iii) That increasing or decreasing an irrelevant alternative may cause the transitivity property of alternatives breached. (iv) That segregating the initial alternative to sub-alternatives may cause the transitivity property of alternatives breached. (v) That eliminating a non-dominated criterion may change the initial rank ordering of alternatives.

In this paper, the characteristic and phenomena of RR in TOPSIS are examined. We can avoid or decrease the RR problem by showing some impact factor in mathematical formulas. Based on a TOPSIS method, we track the outcome of PIS change on any criterion in a applicable case.

## 2. Literature survey

Assume an MCDM case with *m* alternatives *n* criteria, the decision matrix is expressed as

***X***, and which is defined as follows:

$$\boldsymbol{X} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1j} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2j} & \cdots & x_{2n} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ x_{i1} & x_{i2} & \cdots & x_{ij} & \cdots & x_{in} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mj} & \cdots & x_{mn} \end{bmatrix}_{m \times n} \quad (1)$$

where $i$ represents the alternative $i$, $i$=1,2,…,$m$; $j$ denotes the criterion $j$, $j$=1,2,…,$n$; $x_{ij}$ represents the quantitative result of the $j^{th}$ criterion in the $i^{th}$ alternative. The weighted-normalized matrix $\boldsymbol{V}$ is defined as follows:

$$\boldsymbol{V}=\begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1j} & \cdots & v_{1n} \\ v_{21} & v_{22} & \cdots & v_{2j} & \cdots & v_{2n} \\ \vdots & \vdots & \cdots & & \cdots & \vdots \\ v_{i1} & v_{i2} & \cdots & v_{ij} & \cdots & v_{in} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ v_{m1} & v_{m2} & \cdots & v_{mj} & \cdots & v_{mn} \end{bmatrix}_{m\times n} = \begin{bmatrix} r_{11}w_1 & r_{12}w_2 & \cdots & r_{1j}w_j & \cdots & r_{1n}w_n \\ r_{21}w_1 & r_{22}w_2 & \cdots & r_{2j}w_j & \cdots & w_{2n}w_n \\ \vdots & \vdots & \cdots & & \cdots & \vdots \\ r_{i1}w_1 & r_{i2}w_2 & \cdots & r_{ij}w_j & \cdots & r_{in}w_n \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ r_{m1}w_1 & r_{m2}w_2 & \cdots & r_{mj}w_j & \cdots & r_{mn}w_n \end{bmatrix}_{m\times n} \quad (2)$$

where $v_{ij}$ represents the weighted-normalized value of the $j^{th}$ criterion in the $i^{th}$ alternative, $i$=1,2,…,$m$; $j$=1,2,…,$n$. The $r_{ij}$ denotes the normalized value of the $j^{th}$ criterion in the $i^{th}$ alternative, and which is expressed as follows:

$$r_{ij}=\frac{x_{ij}}{\sqrt{\sum_{i=1}^{m}x_{ij}^2}},\ i=1,2,\ldots,m;\ j=1,2,\ldots,n. \quad (3)$$

The $w_j$ represents the weight value of the $j^{th}$ criterion, and $\sum_{j=1}^{n}w_j=1$.

$$v_{ij}=r_{ij}w_j,\ i=1,2,\ldots,m;\ j=1,2,\ldots,n. \quad (4)$$

Determine the positive ideal solution (PIS) and negative ideal solution (NIS) respectively:

$$\text{PIS}=\left(v_1^+,v_2^+,\ldots,v_n^+\right),\ \text{where } v_j^+=\max_i\ v_{ij} \quad (5)$$

$$\text{NIS}=\left(v_1^-,v_2^-,\ldots,v_n^-\right),\ \text{where } v_j^-=\min_i\ v_{ij} \quad (6)$$

The separation measure of Euclidean distance of alternative $i$ from PIS and NIS is defined respectively as follows:

$$S_i^+=\sqrt{\sum_{j=1}^{n}\left(v_j^+-v_{ij}\right)^2} \quad (7)$$

$$S_i^-=\sqrt{\sum_{j=1}^{n}\left(v_{ij}-v_j^-\right)^2} \quad (8)$$

The relative closeness to the ideal solution is defined as follows:

$$C_i = \frac{S_i^-}{S_i^+ + S_i^-}, \text{ for } i=1,2,\ldots,m. \tag{9}$$

The relative closeness of alternatives are ranked in descending order. The smaller the relative closeness, the more front rank is the alternative.

One reason of RR is normalization processes. The Euclidean distance (the $L$=2 metric) is used in TOPSIS. Another reason of RR is the location of PIS or NIS being shifted when an extreme alternative is added or removed. This form of shifted may affect the original rank because of rescaling the performance of each alternative. Some authors tried to use the concept of absolute distances to resolve the RR problem. Kong (2011) used the maximum-minimum way of linear normalization and selected the existing non-dominated alternative to preserve the original rank. García -Cascales and Lamata (2012) used the maximum way of linear normalization and added extreme fictitious alternatives to avoid RR . Senouci et al. (2016) used the maximum-minimum way of linear normalization and provided a dynamic process for keeping the maximum-minimum values for dependable alternatives' performance within the range. de Farias and Ferreira (2019) used the maximum-minimum way of linear normalization and flexibly selected the extreme points in the decision matrix to decrease RR.

The final step in TOPSIS is computing the relative distance to the PIS and NIS respectively and then ranking the alternatives in the descending order. It is calculated by the Euclidean distances, which is also one reason of RR. Some experts tried to redefine the relative closeness. Ren et al. (2007) proposed a M-TOPSIS method. He reallocated alternatives in the separation plan $\left(S_i^+, S_i^-\right)$, and redefined the relative ratio of alternatives in increasing order. Li (2009) proposed a new ranking index and redefined the relative ratio of alternatives in descending order. Kuo (2017) detected the weights of criterion might affect the rank of alternatives when the relative closeness was calculated by examining different rank indices. But it can't solve RR problem in this method. Mufazzal and Muzakkir (2018) proposed a weighted proximity index and related overall proximity value of relative ratio of alternatives to avoid RR. However, it is only applicable to existing non-dominated alternatives. defarias and Ferreira (2019) proposed a R-TOPSIS method to avert RR by using linear normalization and selecting extreme PIS and NIS in the DMs. Yang (2020) proposed a NR-TOPSIS method to avoid RR by rechoosing PIS and NIS in the historical information. Tiwari and Kumar (2021) used a cumulative Gaussion normal probability density function in normalization procedure to avert RR. It is called G-TOPSIS method.

## 3. The Analysis and Predication of RR by Mathematical Viewpoint in TOPSIS and simplified TOPSIS

According to the previous literature review, one reason of RR is the PIS or NIS being changed, i.e., an extreme alternative with minimum or maximum is removed or added. In this section, we concentrate on the mathematical viewpoint of relative closeness and ranking index in TOPSIS by linear normalization method to predicate the condition of RR. The difference between TOPSIS and simplified TOPSIS is above mentioned normalization method. We suppose the situation only on the PIS changes.

Presented with any two alternatives, $A_g$ and $A_h$, the respectively relative closeness of both alternatives are $C_g = \frac{S_g^-}{S_g^+ + S_g^-}$ and $C_h = \frac{S_h^-}{S_h^- + S_h^+}$. Assume that alternative $A_g$ is better than alternative $A_h$, and therefore the $C_g$ will be larger than $C_h$. According to the definition in TOPSIS method, the inequality will be :

$$\frac{S_g^-}{S_g^+ + S_g^-} > \frac{S_h^-}{S_h^+ + S_h^-} \tag{10}$$

After moving the terms, the above inequality is obtained as:

$$S_g^- S_h^+ - S_g^+ S_h^- > 0 \tag{11}$$

The above expression is the fundamental formula judging whether alternative $A_g$ is better than alternative $A_h$. On the flip side, alternative $A_g$ is worse than alternative $A_h$ if the inequality is less than zero. In this situation, the RRP occurs between alternative $A_g$ and $A_h$.

We represent the inequality for analyzing the parameters of RR in the relative closeness. According the formulae of separation measures in TOPSIS, $S_g^+ = \sqrt{\sum_{j=1}^{n} \left(v_j^+ - v_{gj}\right)^2}$ , $S_h^+ = \sqrt{\sum_{j=1}^{n} \left(v_j^+ - v_{hj}\right)^2}$ , $S_g^- = \sqrt{\sum_{j=1}^{n} \left(v_{gj} - v_j^-\right)^2}$ , $S_h^- = \sqrt{\sum_{j=1}^{n} \left(v_{hj} - v_j^-\right)^2}$ , where $j = 1, \ldots, n \in$ criteria. The variables of $v_j^+$ and $v_j^-$ are respectively defined as the maximum and minimum values of $j^{th}$ benefit criterion.

Assume that the value of $j^{th}$ criterion alters, it may result in its $v_j^+$ and $v_j^-$ shift to $v_j^{'+}$ and $v_j^{'-}$ respectively. The related separation measures$(S)$ and relative closeness$(C)$ of alternative $A_g$ and alternative $A_h$ are defined respectively as $S_g^{'+}$ , $S_g^{'-}$, $C_g^{'}$, and $S_h^{'+}$, $S_h^{'-}$, $C_h^{'}$ . The mentioned Eq.(11) is converted as :

$$S_g^{'-} S_h^{'+} - S_g^{'+} S_h^{'-} > 0 \tag{12}$$

The above equation provides a discrimination for judging whether the RR take place or not. If $S_g^{'-} S_h^{'+} - S_g^{'+} S_h^{'-} > 0$, the mentioned of $j^{th}$ criterion alters will not affect the original rank of alternative $A_g$ and $A_h$. In other words, no RR happens. Nevertheless, RR occurs if $S_g^{'-} S_h^{'+} - S_g^{'+} S_h^{'-} < 0$ .

In order to analyze and predicate the RRP status, it is necessary to illustrate the condition of some variables in the related separation measures ($S$)

The function of $x_{ij}$ is defined as the performance of criterion *j* of alternative *i* in the original TOPSIS evaluation, where variable *i* is from 1 to *m*, and variable *j* is from 1 to *n*. The mathematical properties of *m* and *n* belong to constant. The function of $x_j^-$ and $x_j^+$ are respectively defined as the minimum and maximum values on benefit criterion *j*. In this paper, we try to discuss the relations between the alteration of PIS and RR. To focus on RR effects in different PIS, we refer to a method of linear normalization, the min-max normalization, but neglect the minimum value. This normalization method is called as maximum normalization. In addition, the TOPSIS is called as simplified TOPSIS on this situation, and some variables are given as follows.

$$x' = \frac{x}{max(x)} \tag{13}$$

where $x$ is an original value, $x'$ is the normalized value.

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Assume the criterion $l$ of alternative $A_g$ is changed to be new PIS, the ratio of new PIS to original PIS is defined as follows.

$$\lambda_j^+ = \frac{x_j^{'+}}{x_j^+} \tag{14}$$

where $\lambda_j^+ \geq 1$ and $j$=$l$.

Assume that every criterion has equal weight, and belongs to benefit criterion. The ratio of new normalized PIS to original normalized PIS is obtained as follows.

$$v_j^{'+} = v_j^+ \tag{15}$$

where $j$=$l$.

The ratio of new normalized NIS to original normalized NIS is obtained as follows.

$$v_j^{'-} = \frac{v_j^-}{\lambda_j^+} \tag{16}$$

where $j$=$l$.

According the above equation, the ratio of original normalized value to new normalized value on criterion $j$ of alternative $i$ is obtained as follows.

$$v_{ij}^{'} = \frac{v_{ij}}{\lambda_j^+} \tag{17}$$

where $i$=1 to $m$, but $i \neq k$, $j$=$l$.

$$v_{ij}^{'} = v_{ij} \tag{18}$$

where $i$=$k$, $j$=$l$.

In case of PIS changes on $l^{th}$ criterion, four original measures to PIS and NIS of alternative $A_g$ and alternative $A_h$ are changed and there are defined as $S_g^{'+}$, $S_h^{'+}$, $S_g^{'-}$ and $S_h^{'-}$ as follows.

$$S_g^{'+} = \sqrt{\sum_{j=1}^{n} \left( v_j^{'+} - v_{gj}^{'} \right)} \tag{19}$$

$$S_h^{'+} = \sqrt{\sum_{j=1}^{n} \left( v_j^{'+} - v_{hj}^{'} \right)} \tag{20}$$

$$S_g^{'-} = \sqrt{\sum_{j=1}^{n} \left( v_{gj}^{'} - v_j^{'-} \right)} \tag{21}$$

$$S_h^{'-} = \sqrt{\sum_{j=1}^{n} \left( v_{hj}^{'} - v_j^{'-} \right)} \tag{22}$$

Consequently, the Eq.(12) will be represented as follows.

$$\sqrt{S_g^{-2} + \left( \left( \frac{1}{\lambda_l^+} \right)^2 - 1 \right) \cdot \left( v_{gl} - v_l^- \right)^2} \cdot \sqrt{S_h^{+2} + \left( v_l^+ - \frac{v_{hl}}{\lambda_l^+} \right)^2 - \left( v_l^+ - v_{hl} \right)^2} - \sqrt{S_g^{+2} + \left( v_l^+ - \frac{v_{gl}}{\lambda_l^+} \right)^2 - \left( v_l^+ - v_{gl} \right)^2} \cdot \sqrt{S_h^{-2} + \left( \left( \frac{1}{\lambda_l^+} \right)^2 - 1 \right) \cdot \left( v_{hl} - v_l^- \right)^2} > 0 \tag{23}$$

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In order to demonstrate the strong correlation between the $\lambda_l^+$ and the Eq.(23), a simpler variable, $\delta$, is defined as the difference between both sides in the Eq.(24).

$$\delta = \alpha - \beta \tag{24}$$

Where

$$\alpha = \sqrt{S_g^{-2} + \left(\left(\frac{1}{\lambda_l^+}\right)^2 - 1\right) \cdot \left(v_{gl} - v_l^-\right)^2} \cdot \sqrt{S_h^{+2} + \left(v_l^+ - \frac{v_{hl}}{\lambda_l^+}\right)^2 - \left(v_l^+ - v_{hl}\right)^2} \tag{25}$$

$$\beta = \sqrt{S_g^{+2} + \left(v_l^+ - \frac{v_{gl}}{\lambda_l^+}\right)^2 - \left(v_l^+ - v_{gl}\right)^2} \cdot \sqrt{S_h^{-2} + \left(\left(\frac{1}{\lambda_l^+}\right)^2 - 1\right) \cdot \left(v_{hl} - v_l^-\right)^2} \tag{26}$$

The performance of alternative $A_g$ is better than alternative $A_h$ on any variation of the relevant parameters if $\delta \geq 0$. In other words, no RR occurs between alternative $A_g$ and alternative $A_h$. However, the RR happens when $\delta < 0$. The alternative $A_h$ is better than alternative $A_g$ in this condition. We try to appraise and predicate the timing of RR by changing PIS. Accordingly, the PIS being higher, the value of $\delta$ being lower. The RR occurs when $\delta < 0$. In this case namely, the Eq. (23) can predicate the timing of RR. The Example 1 will be illustrated about above assumption.

**Example 1**: A four alternatives with three criteria example

The fighter selection example of Hwang and Yoon (1981) is used and represented as shown in Table1.

**Table 1**
Four alternatives and three criteria in the decision matrix

| Alternative | Criteria | | | Note |
|---|---|---|---|---|
| | $X_1$ | $X_2$ | $X_3$ | |
| $A_1$ | 2.0 | 1,500 | 20,000 | All criteria belong to benefit |
| $A_2$ | 2.5 | 2,700 | 18,000 | |
| $A_3$ | 1.8 | 2,000 | 21,000 | |
| $A_4$ | 2.2 | 1,800 | 20,000 | |

Firstly, the weight of each criterion is supposed to be equal. In the second, a linear normalization method, maximum normalization, is used to execute and calculate the ranks of four alternatives as shown in Table2.

**Table 2**
The rank of four alternatives

| Alternative | Linear normalization | | | |
|---|---|---|---|---|
| | $S_i^+$ | $S_i^-$ | $C_i$ | Rank |
| $A_1$ | 0.163231 | 0.041460 | 0.202549 | 4 |

| | | | | |
|---|---|---|---|---|
| $A_2$ | 0.047619 | 0.175097 | 0.786189 | 1 |
| $A_3$ | 0.127199 | 0.077961 | 0.390003 | 2 |
| $A_4$ | 0.119154 | 0.072277 | 0.377563 | 3 |

In Table 2, the ranks of four alternatives are $A_2 > A_3 > A_4 > A_1$. For verifying the Eq. (23), an alternative $A_5$ is added, and its benefit of each criterion are the highest value among the [$X_1$, $X_2$, $X_3$]. They are [2.5, 2,700, 21,000], and five alternatives with three criteria are shown in Table3.

**Table 3**
Five alternatives and three criteria in the decision matrix

| Alternative | Criteria $X_1$ | $X_2$ | $X_3$ | Note |
|---|---|---|---|---|
| $A_1$ | 2.0 | 1,500 | 20,000 | All criteria are belong to benefit |
| $A_2$ | 2.5 | 2,700 | 18,000 | |
| $A_3$ | 1.8 | 2,000 | 21,000 | |
| $A_4$ | 2.2 | 1,800 | 20,000 | |
| $A_5$ | 2.5 | 2,700 | 21,000 | |

The new ranks are shown in Table 4. No RR happens when the alternative $A_5$ is added, and the ranks of five alternatives are $A_5 > A_2 > A_3 > A_4 > A_1$. However, the RR occurs if the criterion $X_2$ value of new alternative, $A_5$, is gradually increased. We respectively specify $A_3$ to be $A_g$, and $A_4$ to be $A_h$, to be a better similarity of the Eg.(23).

**Table 4**
The rank of five alternatives

| Alternative | Linear normalization $S_i^+$ | $S_i^-$ | $C_i$ | Rank |
|---|---|---|---|---|
| $A_1$ | 0.163231 | 0.041460 | 0.202549 | 5 |
| $A_2$ | 0.047619 | 0.175097 | 0.786189 | 2 |
| $A_3$ | 0.127199 | 0.077961 | 0.38Th0003 | 3 |
| $A_4$ | 0.119154 | 0.072277 | 0.377563 | 4 |
| $A_5$ | 0.000000 | 0.181457 | 1.000000 | 1 |

According the definition of Eq.(14), the $\lambda_2^+$ will be larger than 1.0 when the value of original PIS is shifted upward. For example, if the value of criterion $X_2$ is shifted upward to 5,400, double value of the original one, $\lambda_2^+$ will be changed to 2 from 1. It is also noted that $\lambda_2^+ = 2$. We try to change the value of $\lambda_2^+$, from 1.0 to 2.0, by increasing criterion $X_2$ of $A_5$. Based on obtained data in Table 5, the RR happens between $A_3$ and $A_4$ when $\lambda_2^+ = 1.2$. At the same time, $\delta < 0$ ($\delta = -0.000143$). It means that
$\delta$ is a threshold value in predicating the timing of RR. The effect of $\lambda_2^+$ are demonstrated in Table 5.

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**Table 5**
The effect of $\lambda_2^+$ on rank reversal

| $\lambda_2^+$ | $S'^+_3$ | $S'^+_4$ | $S'^-_3$ | $S'^-_4$ | $\delta$ | RR occurs | Rank of $A_3$ and $A_4$ |
|---|---|---|---|---|---|---|---|
| 1.0 | 0.127199 | 0.119154 | 0.077961 | 0.072277 | 0.000096 | No | $A_3$>$A_4$ |
| 1.1 | 0.143398 | 0.138185 | 0.073598 | 0.070611 | 0.000045 | No | $A_3$>$A_4$ |
| 1.2 | 0.158069 | 0.154272 | 0.070098 | 0.069317 | –0.000143 | Yes | $A_4$>$A_3$ |
| 1.3 | 0.171098 | 0.167998 | 0.067248 | 0.068293 | –0.000387 | Yes | $A_4$>$A_3$ |
| 1.4 | 0.182619 | 0.179828 | 0.064897 | 0.067469 | –0.000651 | Yes | $A_4$>$A_3$ |
| 1.5 | 0.192819 | 0.190120 | 0.062937 | 0.066798 | –0.000914 | Yes | $A_4$>$A_3$ |
| 1.6 | 0.201882 | 0.199150 | 0.061286 | 0.066243 | –0.001168 | Yes | $A_4$>$A_3$ |
| 1.7 | 0.209974 | 0.207134 | 0.059884 | 0.065779 | –0.001408 | Yes | $A_4$>$A_3$ |
| 1.8 | 0.217231 | 0.214243 | 0.058682 | 0.065388 | –0.001632 | Yes | $A_4$>$A_3$ |
| 1.9 | 0.223772 | 0.220612 | 0.057646 | 0.065056 | –0.001840 | Yes | $A_4$>$A_3$ |
| 2.0 | 0.229694 | 0.226351 | 0.056747 | 0.064770 | –0.002033 | Yes | $A_4$>$A_3$ |

When $\lambda_2^+$ increase, $S'^+_g$ and $S'^+_h$ also increase, but $S'^-_g$ and $S'^-_h$ decrease on the contrary. Maybe it is difficult to predict RR. However, we can infer the RR condition by $\delta$. In the first and second row of Table 5, $\delta$=0.000096, $\delta$=0.000045 when $\lambda_2^+ = 1.0$, $\lambda_2^+ = 1.1$, respectively. According the Eq.(23) and Eq.(24), no RR occurs at these two conditions. Nonetheless, the value of $\delta$ converts to negative and RR happens when $\lambda_2^+$ is larger than 1.2. We try to increase $\lambda_2^+$ from 1.2 to 2.0, the more negative value of $\delta$ will be. The RR still occurs persistently. Hence, $\lambda_2^+$ can be an important and effective scale for predicating RR. Some people propose that RR could be avoided by adding an extreme alternative. However, there is still no technique or manner to prevent RR through above example and illustration in this paper.

## 4. Summary

The RR condition in TOPSIS has been discussed and derived by mathematical method in this paper. Because of non-dominated criterion, the RR usually happens. In the TOPSIS process of mathematical formula, many parameters will be altered when the value of non-dominated criterion changes. Thus, determining some non-dominated criteria could avoid RR before evaluating the rank of all alternatives. The proposed variable, $\delta$, could serve as a threshold value to predicate RR. That is to say RR occurs when $\delta < 0$, and there is no RR when $\delta > 0$.

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