

Another micro-open set in micro topological spaces

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Abstract. Topologists have introduced a variety of topologies, as is well known. Fine topology, supra topology, ideal topology, fuzzy topology, .nano topology, and .micro topology are the topologies that exist today, with .micro topology being an enlargement of .nano topology, which has been established by Chandrasekar S. We introduce a new weaker type of .micro semi-open set, called .micro S_w -open sets, and discuss some of its features in this work. Compare it to some other .micro-open sets as well.

Keywords. Mic topological spaces, Mic S_w -open sets, Mic semi-open set, Mic β -open set, Mic open set

1. Introduction

Since known topology is one of the most important areas of mathematics, the idea of topology is getting broader and growing day by day. Many types of topologies have been created to date since Kelly introduced bitopological spaces in 1963 [1]. The study of fuzzy topological spaces introduced by Chang in 1968 [2]. The concept of the ideal in topological space was introduced by Kuratowski. Since 2011, Shabir and Naz introduced the notion of soft topological spaces defined over an initial universe with a fixed set of parameters [3]. Powar and Rajak introduced fine topological spaces in 2012 [4]. Introduced by Thivagar and Richard in 2013, the notion of nano topology is an extension of set theory for the study of intelligent systems characterized by sufficient and incomplete information [5]. Micro topology was a new idea constructed from nano*topology by Chandrasekar [6] in 2019. Lower approximations produce subsets with known elements that would constitute an interesting subset, while the higher approximation produces subsets with uncertain objects that may or may not constitute an interesting subset [7]. In addition, Chandrasekar and Swathi 2019 constructed mic*semi-open and pre-open sets [8]. The term mic open set was introduced by Jassim, Rasheed and Faris in 2021 [9]. Also, in 2020, Ibrahim introduced and analyzed another open set in $(V, \tau_R(X), \mu_R(x))$ called mic-open set and described a new generalized closed set called mic - g - closed and gave the analyzed behaviour of this set [10, 11]. In this article, we develop a new type of mic open set called Mic-Open, examine some of its attributes and characterizations, and compare it to other types of open sets. Throughout this research mic represent the micro.

2. Preliminaries

This section is to give some results that we used in the next sections.

Definitions 2.1: [7]

Let V be the universe, which is a non-empty finite collection of things, and R be the indiscernibility relation, which is an equivalence relation on V . The pair (V, R) is said to be the approximation space. Let $X \subseteq V$:

- (i) Denote $L_R(X) = \bigcup_{x \in V} \{R(x) : R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by x , the lower approximation of X with respect to R .
- (ii) Denote $U_R(X) = \bigcup_{x \in V} \{R(x) : R(x) \cap X \neq \phi\}$, the upper approximation of X with respect to R .
- (iii) Denote $B_R(X) = U_R(X) - L_R(X)$ the boundary region of X with respect to R .

Definitions 2.2: [5] Let V be the universe set, and R be an equivalence relation on V and $\tau_R(X) = \{V, \phi, L_R(X), U_R(X), B_R(X)\}$, where $X \subseteq V$. If $\tau_R(X)$ satisfies the following axioms:

- (i) $V, \phi \in \tau_R(X)$.
- (ii) The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.
- (iii) The intersection of the elements of any finite subcollection $\tau_R(X)$ is in $\tau_R(X)$.

Then $\tau_R(X)$ is a topology on V called the nano topology with respect to X we call $(V, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called nano open sets and $[\tau_R(X)]^c$ is called the dual nano topology of $\tau_R(X)$.

Theorem 2.3: [6] If $(V, \tau_R(X))$ is a nano topological space with respect to X , where $X \subseteq V$ and A, B are subsets of V , then $\mu_R(X) = \{N \cup (N' \cap \mu) : N, N' \in \tau_R(X) \text{ and } \mu \notin \tau_R(X)\}$ is called the mic topology on V with respect to X . The triple $(V, \tau_R(X), \mu_R(X))$ is called mic topological space and the elements of $\mu_R(X)$ are called mic - open sets and its complement called a mic - closed set

Definition 2.4: [10] Let $(V, \tau_R(X), \mu_R(X))$ be a space. Then the mic closure of a set A denoted by $mic - cl(A)$ and defined by $mic - cl(A) = \bigcap \{F : F \text{ is mic closed set in } U \text{ and } A \subseteq F\}$. The mic interior of A is denoted by $mic - int(A)$ and defined as $mic - int(A) = \bigcup \{G : G \text{ is mic - open set and } G \subseteq A\}$.

Definition 2.5: [10] For any two sets A and B in $(U, \tau_R(X), \mu_R(X))$, we have:

1. $A \in (\mu_R(X))^c$ if and only if $mic - cl(A) = A$.
2. $A \in \mu_R(X)$ if and only if $mic - int(A) = A$.
3. $A \subseteq B$ implies $mic - int(A) \subseteq mic - int(B)$ and $mic - cl(A) \subseteq mic - cl(B)$.
4. $mic - cl(mic - cl(A)) = mic - cl(A)$ and $mic - int(mic - int(A)) = mic - int(A)$.
5. $mic - cl(A \cup B) \supseteq mic - cl(A) \cup mic - cl(B)$.
6. $mic - cl(A \cap B) \subseteq mic - cl(A) \cap mic - cl(B)$.
7. $mic - int(A \cup B) \supseteq mic - int(A) \cup mic - int(B)$.
8. $mic - int(A \cap B) \subseteq mic - int(A) \cap mic - int(B)$.
9. $mic - cl(A^c) = [mic - int(A)]^c$.
10. $mic - int(A^c) = [mic - cl(A)]^c$.

Definition 2.6: Let $(V, \tau_R(X), \mu_R(X))$ be a space and $A \subseteq V$. Then:

1. A is mic semi-open set if $A \subseteq mic - cl(mic - int(A))$ [6]
2. A is mic pre-open set if $A \subseteq mic - int(mic - cl(A))$ [6]

3. A is θ -mic-open set if for each $x \in A$, there exists a mic - open set G such that $x \in G$ and $mic - cl(G) \subseteq A$. [9]

Proposition 2.7: [10] In any space $(V, \tau_R(X), \mu_R(X))$:

1. mic - open will be actually mic semi-open.
2. $A \in MicO(V, X) \Rightarrow A \in MicPO(V, X)$.

3. Mic S_w -open sets

Definition 3.1: Let $(V, \tau_R(X), \mu_R(X))$ be a space. Then a subset A of V together with the empty set is called mic S_w -open set if $mic - int(A) \neq \phi$. The family of mic S_w -open sets of V is denoted by $MicS_wO(V, X)$ and named the complementary of mic S_w -open sets mic S_w -closed; that is $mic - cl(A^c) \neq V$. The family of mic S_w -closed sets of V is denoted by $MicS_wC(V, X)$.

Example 3.2: Consider $V = \{p, q, r, s, t\}$, $V/R = \{\{p\}, \{q, r, s\}, \{t\}\}$ and $X = \{p, q\} \subseteq V$. Then $\tau_R(X) = \{V, \phi, \{p\}, \{q, r, s\}, \{p, q, r, s\}\}$ is a nano topology on V with respect to X . Now if $\mu = \{t\}$, then $\mu_R(X) = \{V, \phi, \{p\}, \{t\}, \{p, t\}, \{q, r, s, t\}, \{q, r, s\}, \{p, q, r, s\}\}$ is a mic topology on V . Then here $\{p, t, s\}$ is a mic S_w -open sets, because $mic - int(\{p, t, s\}) = \{p, t\} \neq \phi$

Proposition 3.3: Every non-empty mic semi-open subset is mic S_w -open set in any mic topological space.

Proof: Let $A \neq \phi$ be a mic semi-open set. Then by [Definition 2.6] $A \subseteq mic - cl(mic - int(A))$, then $mic - int(A) \neq \phi$, because if $mic - int(A) = \phi$ this implies that $mic - cl(mic - int(A)) = \phi$, but $A \subseteq mic - cl(mic - int(A))$ which get a contradiction. Thus A is mic S_w -open set.

As an illustration, the opposite of the preceding proposition is not generally true:

Example 3.4: Let $V = \{1, 2, 3, 4\}$, $V \setminus R = \{\{1, 2\}, \{3\}, \{4\}\}$ and $X = \{2, 3, 4\}$. Then $\tau_R(X) = \{V, \phi, \{3\}, \{4\}, \{1, 2\}, \{1, 2, 3\}, \{1, 2, 4\}, \{3, 4\}\}$ and if $\mu = \{2, 4\}$, then the mic topology on V with respect to X is $\mu_R(X) = \{V, \phi, \{2\}, \{3\}, \{4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2\}, \{1, 2, 3\}, \{1, 2, 4\}, \{2, 3, 4\}\}$. Then here the family of all mic semi-open set is $MicSO(V, X) = \mu_R(X)$ and then $\{1, 3, 4\}$ is mic S_w -open set since $mic - int(\{1, 3, 4\}) = \{3, 4\} \neq \phi$, but not a mic semi-open set.

Proposition 3.5: Let $(V, \tau_R(X), \mu_R(X))$ be a mic topological space and $A \subseteq V$. Then A is mic S_w -open set if and only if contains a mic semi-open set.

Proof: Let A be mic S_w -open set. Then $mic - int(A) \neq \phi$, as it is clear that $mic - int(A) \subseteq A$ and $mic - int(A)$ is a mic -open set implies that $mic - int(A)$ is a mic semi-open set. Thus A contains a mic semi-open set.

Conversely: Let B be a mic semi-open subset of U such that $B \subseteq A$. Then $mic - int(A) \subseteq mic - int(B)$, now $mic - int(B) \neq \phi$ implies that $mic - int(A) \neq \phi$ this implies that A is mic S_w -open set.

Definition 3.6: A set B in nano space $(V, \tau_R(X))$ is called nano S_w -open set if $Nint(B) \neq \phi$.

Proposition 3.7: Every nano S_w -open set is mic S_w -open.

Proof: Let A be a nano S_w -open set. Then $Nint(A) \neq \phi$, and since every nano open set is a mic - open set this implies that $Nint(A) \subseteq mic - int(A)$ and then $mic - int(A) \neq \phi$, and then A is mic S_w -open set.

For generally, the reverse of the preceding proposition may not always be correct, as illustrated in [Example 3.4] Set $\{2\}$ is a mic S_w -open set, not really nano S_w -open set.

Lemma 3.8: The union of a family of mic S_w -open sets is mic S_w -open.

Proof: Let $\{A_i; i \in \Delta\}$ be a family of mic S_w -open sets in mic topological space $(V, \tau_R(X), \mu_R(X))$. Then for each $i \in \Delta$, $mic - int(A_i) \neq \phi$, and since for each $j \in \Delta, A_j \subseteq \bigcup_{i \in \Delta} A_i$ this implies that $mic - int(A_j) \subseteq mic - int(\bigcup_{i \in \Delta} A_i) \neq \phi$. Thus $\bigcup_{i \in \Delta} A_i$ is mic S_w -open set in V .

The intersection of two mic S_w -open sets may not be mic S_w -open as shown in [Example 3.2] the subsets $A = \{p, q\}$ and $B = \{q, r, s, t\}$ are mic S_w -open sets but $A \cap B = \{q\}$ is not mic S_w -open.

Corollary 3.9: The intersection of a family of mic S_w -closed set is mic S_w -closed.

Proof: Clear.

Remark 3.10: The concepts of mic S_w -open set and mic pre-open set are independent as an example below:

Example: Let $U = \{1,2,3,4\}$, $U \setminus R = \{\{1\}, \{3\}, \{2, 4\}\}$ and $X = \{2,4\}$. Then the nano topology on U with respect to X is $\tau_R(X) = \{U, \phi, \{2,4\}\}$ and if $\mu = \{1\}$, then $\mu_R(X) = \{U, \phi, \{1\}, \{2,4\}, \{1,2,4\}\}$. Then here $\{4\}$ is mic pre-open set but not mic S_w -open and $\{2,3,4\}$ is mic S_w -open set but not mic pre-open set.

Theorem 3.11: The mic closure of each non-empty mic S_w -open set in mic space is also a mic S_w -open set.

Proof: Let A be any mic S_w -open set in $(V, \tau_R(X), \mu_R(X))$. Then $mic - int(A) \neq \phi$, and now since $A \subseteq mic - cl(A)$ and $A \neq \phi$ this implies that $mic - int(mic - cl(A)) \neq \phi$, and then $mic - cl(A)$ is mic S_w -open set.

Proposition 3.12: Let $A \subseteq V$ be a non-empty mic pre-open set in $(V, \tau_R(X), \mu_R(X))$. Then $mic - cl(A) \in MicS_wO(V, X)$.

Proof: Let $A \neq \phi$ be a mic pre-open set. Then $A \subseteq mic - int(mic - cl(A))$ and since $A \neq \phi$ implies that $mic - cl(A) \neq \phi$ and then $mic - int(mic - cl(A)) \neq \phi$, therefore; $mic - cl(A)$ is mic S_w -open set.

Remark 3.13: The complement of mic S_w -open set may not be mic S_w -open in space $(V, \tau_R(X), \mu_R(X))$. As exist in [Example 3.2] the set $\{p, r, s, t\}$ is mic S_w -open set but $\{p, r, s, t\}^c = \{q\}$ which is not mic S_w -open set.

Definition 3.11: Let $(V, \tau_R(X), \mu_R(X))$ be a space. Then the union of all mic S_w -open sets contained in $A \subseteq V$, is called mic S_w -the interior of A and denoted by $Mic - S_wint(A)$, so it is the largest mic S_w -open subset of A . The intersection of all mic S_w -closed set which containing A is called mic S_w -closure of A and denoted by $Mic - S_wcl(A)$, so it is the smallest mic S_w -closed set containing A .

Theorem 3.12: For any subsets A and B of a mic topological space $(V, \tau_R(X), \mu_R(X))$, we have:

1. $A \in MicS_wO(V, X)$ if and only if $Mic - S_wint(A) = A$.
2. $A \in MicS_wC(V, X)$ if and only if $Mic - S_wcl(A) = A$.
3. If $A \subseteq B$, then $Mic - S_wint(A) \subseteq Mic - S_wint(B)$ and $Mic - S_wcl(A) \subseteq Mic - S_wcl(B)$.
4. $Mic - S_wint(A) \cup Mic - S_wint(B) \subseteq Mic - S_wint(A \cup B)$.
5. $Mic - S_wint(A \cap B) \subseteq Mic - S_wint(A) \cap Mic - S_wint(B)$.
6. $Mic - S_wcl(A) \cup Mic - S_wcl(B) \subseteq Mic - S_wcl(A \cup B)$.
7. $Mic - S_wcl(A \cap B) \subseteq Mic - S_wcl(B) \cap Mic - S_wcl(A \cup B)$.

8. $Mic - S_w int(A^c) = [Mic - S_w cl(A)]^c$.
9. $Mic - S_w cl(A^c) = [Mic - S_w int(A)]^c$.
10. $V \setminus Mic - S_w cl(V \setminus A) = Mic - S_w int(A)$.
11. $V \setminus Mic - S_w int(V \setminus A) = Mic - S_w cl(A)$.

Definition 3.13: A non-empty subset A of a mic space $(V, \tau_R(X), \mu_R(X))$ is called mic dense if $mic - cl(A) = V$.

Definition 3.14: A space $(V, \tau_R(X), \mu_R(X))$ is called mic hyperconnected space if every non-empty mic - open subset of U is mic - dense.

Theorem 3.15: If $(V, \tau_R(X), \mu_R(X))$ is mic hyperconnected, then $MicS_w O(V, X) = MicSO(V, X)$.

Proof: Let $A \in MicS_w O(V, X)$. Then $mic - int(A) \neq \phi$ and since V is mic hyperconnected space implies that $mic - cl(mic - int(A)) = U$, but $A \subseteq V = mic - cl(mic - int(A))$ and then A is mic semi-open set, this implies that $MicS_w O(V, X) \subseteq MicSO(V, X)$, while by **[Proposition 3.3]** $MicSO(V, X) \subseteq MicS_w O(V, X)$; therefore, $MicS_w O(V, X) = MicSO(V, X)$.

Definition 3.16: A mic topological space $(V, \tau_R(X), \mu_R(X))$ is called mic locally indiscrete if every mic - open set is mic - closed.

Theorem 3.17: If A is a mic S_w -open subset of a mic locally indiscrete hyperconnected space $(V, \tau_R(X), \mu_R(X))$, then A is a mic - open set.

Proof: Let A be mic S_w -open set in V . Then $mic - int(A) \neq \phi$, and since V is mic hyperconnected space, then A is mic semi-open set by **[Theorem 3.15]**, implies that $A \subseteq mic - cl(mic - int(A))$ and since V is mic locally indiscrete, then $mic - cl(mic - int(A)) = mic - int(A)$; therefore, $A \subseteq mic - int(A)$ implies that A is a mic-open set in V .

Corollary 3.18: Every mic S_w -open set in a mic locally indiscrete hyperconnected space $(U, \tau_R(X), \mu_R(X))$ is a mic pre-open set.

Proof: Follows from **[Theorem 3.17]** and **[Proposition 2.7]**.

Lemma 3.19: A subset A of a mic space $(V, \tau_R(X), \mu_R(X))$ is mic S_w -open set if and only if, for each $x \in A$, there exists a mic S_w -open set B such that $x \in B \subseteq A$.

Lemma 3.20: If A is a θ - mic open set in mic space $(V, \tau_R(X), \mu_R(X))$, then A is mic S_w -open set.

Proof: Let A be θ - mic open set, then by **[Definition 2.8 (3)]** for each $x \in A$ there exists a mic - open set G such that $x \in G \subseteq mic - cl(G) \subseteq A$ and then $mic - int(A) \neq \phi$ this implies that A is mic S_w -open set.

Lemma 3.21: Let $(V, \tau_R(X), \mu_R(X))$ be a mic space. Then:

1. If $A \in MicS_w O(V, X)$ and B be any subset of V , then $A \cup B \in MicS_w O(V, X)$.
2. The intersection of any set with a non-void mic S_w -closed set is in $MicS_w C(V, X)$.

Proof: (1) it is clear.

(2) Let A and B be any two subsets of V such that $B \neq \phi$ and be in $MicS_w C(V, X)$.

Then $X \setminus B \in MicS_w O(V, X)$, this implies that $mic - int(X \setminus B) \neq \phi$. Now we have to show that $X \setminus (A \cap B)$ is mic S_w -open set, and then

$$mic - int(X \setminus A) \cup mic - int(X \setminus B) \subseteq mic - int((X \setminus A) \cup (X \setminus B)) = mic - int(X \setminus (A \cap B))$$

But since $mic - int(X \setminus B) \neq \phi$, then $mic - int(X \setminus (A \cap B)) \neq \phi$ this implies that $X \setminus (A \cap B) \in MicS_wO(V, X)$, thus $A \cap B \in MicS_wC(V, X)$.

Remark 3.22: Let Y be a subspace of the mic space V . Now if G is mic S_w -open set in V , then $G \cap Y$ may not be mic S_w -open in Y as illustrated in the following example:

Example: Let $U = \{1, 2, 3, 4\}$, $U \setminus R = \{\{1\}, \{3\}, \{2, 4\}\}$ and $X = \{2, 4\}$. Then the nano topology on U with respect to X is $\tau_R(X) = \{U, \phi, \{2, 4\}\}$ and if $\mu = \{1\}$, then $\mu_R(X) = \{U, \phi, \{1\}, \{2, 4\}, \{1, 2, 4\}\}$, if $Y = \{3, 4\}$, then $\mu_{RY}(X) = \{Y, \phi, \{4\}\}$ is a mic relative topology on Y . Now the $G = \{1, 3\}$ is mic S_w -open in U but $G \cap Y = \{3\}$ which is not mic S_w -open set in Y because it $mic - int_Y(G) = \phi$.

Theorem 3.23: Let Y be a mic subspace of the mic space $(V, \tau_R(X), \mu_R(X))$ and K be any mic S_w -open set in V . If $mic - int(K) \cap Y \neq \phi$, then $K \cap Y \in MicS_wO(Y, X)$.

Proof: Let $mic - int(K) \cap Y \neq \phi$. Then $mic - int(K)$ is a mic S_w -open set in V , this implies that $mic - int(K) \cap Y$ is mic S_w -open set in Y regarding to the mic relative topology.

Proposition 3.24: If Y be a mic subspace of the space $(V, \tau_R(X), \mu_R(X))$ and if $G \subseteq Y$ is a mic S_w -open set in V , then $G \in MicS_wO(Y, X)$.

Proof: Let $G \subseteq Y$ such that $G \in MicS_wO(V, X)$. Then $mic - int_V(G) \neq \phi$, where $mic - int_V(G)$ represent the mic interior of G with respect to the mic topology on V , but always $mic - int_V(G) \subseteq mic - int_Y(G)$, this implies that $mic - int_Y(G) \neq \phi$. Hence G is mic S_w -open set in Y .

The reverse of the above proposition may not be true in general as exist in the example of **Remark 3.22**, Y is mic S_w -open set in itself, but it is not mic S_w -open set in U , because $mic - int(Y) = \phi$.

Conclusion

As is known, many types of topological spaces have been introduced, and one of these topologies that researchers have been working on is called mic topology, which has been raised in recent years and on which many topologists have been working. In this article we introduce the Mic Topological Space, the mic S_w -open Set, a new mic-open set in Mic Topological Space, and provide some characterizations and features for this form of mic sets, as well as a Comparison with existing types of micro open sets.

Conflict of Interest

The authors declare no conflict of interest.

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